1. Sketch the graph of the binary sigmoidal function
$$f(x) = \frac{1}{1+e^{-x}}$$
Verify that:
$$0 < f(x) < 1$$
and
$$f(-x) = 1 - f(x)$$
Calculate $f(0)$, $f(2)$, and $f(-2)$.

2. Convince yourself that $f(x)$, in Problem 1, is bounded, Continuous, monotonically increasing, and differentiable at all points.

3. In problem.1, Verify that
$$x = \ln[f(x)] - \ln[1 - f(x)]$$
What is the value of x when $f(x) = 0.75$?

4. Verify that the derivative of $f(x)$, in problem.1, is expressed as:
$$\frac{df(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2}$$
or alternatively,
$$\frac{df(x)}{dx} = f(x)[1 - f(x)]$$

5] Sketch the graph of $\frac{df(x)}{dx}$, in Problem.4 verify that this graph is symmetric about the $X=0$ axis, and that $\frac{df(x)}{dx}$ has a maximum value of 0.25 at $x=0$. Calculate

$$\left.\frac{df(x)}{dx}\right|_{x=2} \text{ and } \left.\frac{df(x)}{dx}\right|_{x=-2}$$

6] For a given value of $\frac{df(x)}{dx}$, in Problem 4, verify that there are two values of x, x_1 and x_2, such that

$$x_1 + x_1 = 0$$

and

$$f(x_1) + f(x_2) = 1$$

Calculate x_1, x_2, $f(x_1)$, and $f(x_2)$ when $\frac{df(x)}{dx} = 0.15$

7] A neuron has a binary sigmoidal function of the form given in Problem.1. It receives four inputs 0.8, 0.3, 0.2, and 0.6 with weights −0.2, 0.3, 0.8 and 0.5, respectively. Find the neuronal output if the weight bias is 0.25.

8] A neuron has a binary sigmoidal function of the form given in Problem.1. Derive an expression for the activation, y, in terms of the neuronal output, s. Calculate y for $s=0.44$.

9 Sketch the graph of the bipolar sigmoidal function:

$$g(x) = \frac{2}{1 + e^{-x}} - 1$$

Verify that:

$$-1 < g(x) < 1$$

and

$$g(-x) = -g(x)$$

Calculate $g(0)$, $g(2)$, and $g(-2)$.

10 Verify that the bipolar sigmoidal function $g(x)$, in problem 9, is related to the binary sigmoidal function $f(x)$, in problem 1, through the relation

$$g(x) = 2f(x) - 1$$

Represent this relation graphically. What is the value of $g(x)$ when $f(x) = 0.5$? What is the value of $f(x)$ when $g(x) = -1$?

11 Verify that the derivative of $g(x)$, in problem 9, is expressed as:

$$\frac{dg(x)}{dx} = \frac{2e^{-x}}{(1+e^{-x})^2} = 0.5\left[1 - g^2(x)\right]$$

and that it is related to $\frac{df(x)}{dx}$, in problem 4, as

$$\frac{dg(x)}{dx} = 2\frac{df(x)}{dx}$$

sketch the graph of $\frac{dg(x)}{dx}$, referring to its main properties. Calculate:

$$\left.\frac{dg(x)}{dx}\right|_{x=2} \quad \text{and} \quad \left.\frac{dg(x)}{dx}\right|_{x=-2}$$

12] Aneuron has abipolar sigmoidal function of the form given in problem.9. Derive an expression for the activation, y, in terms of the neuronal output, s. Calculate y for $s = 0.6$ and for $s = -0.6$.

13] Aneuron has an activation y and output s. Sketch, on the same s-y coordinate axes, the graphs of $y(s)$, y as a function of s, Considering the following two Cases:

a) The neuronal output is of the binary sigmoidal type, in Problem.1. Verify, in this situation, that

$$y(1-s) = -y(s) \quad , \quad 0 < s < 1$$

b) The neuronal output is of the bipolar sigmoidal type, in Problem -9. Verify, in this situation, that

$$y(-s) = -y(s) \quad , \quad -1 < s < 1$$

(Make use of the expressions derived for $y(s)$ in problem. 8 and 12).

14] Aneuron has abipolar sigmoidal function of the form given in problem.9. It receives three inputs 1.2, 2.2, and 1.6 with weights 0.6, 0.8 and 0.7,

respectively. Find the neuronal output if the weight bias is -1.8.

15 A neuron receives four inputs -2.5, -3.2, -5.1, and 4.4 with weights -1.5, 1.5, -1.8 and -2.1, respectively. The neuronal output is 0.65. Find the weight bias in each of the following two cases:

a) The neuron has a binary sigmoidal function of the form given in problem 1.

b) The neuron has a bipolar sigmoidal function of the form given in problem 9.

16 Consider the neural network illustrated in Fig. 1. Determine the signals S_1 and S_2 in each of the following two cases:

a) The output neurons have a binary sigmoidal function of the form given in problem 1.

b) The output neurons have a bipolar sigmoidal function of the form given in problem 9.

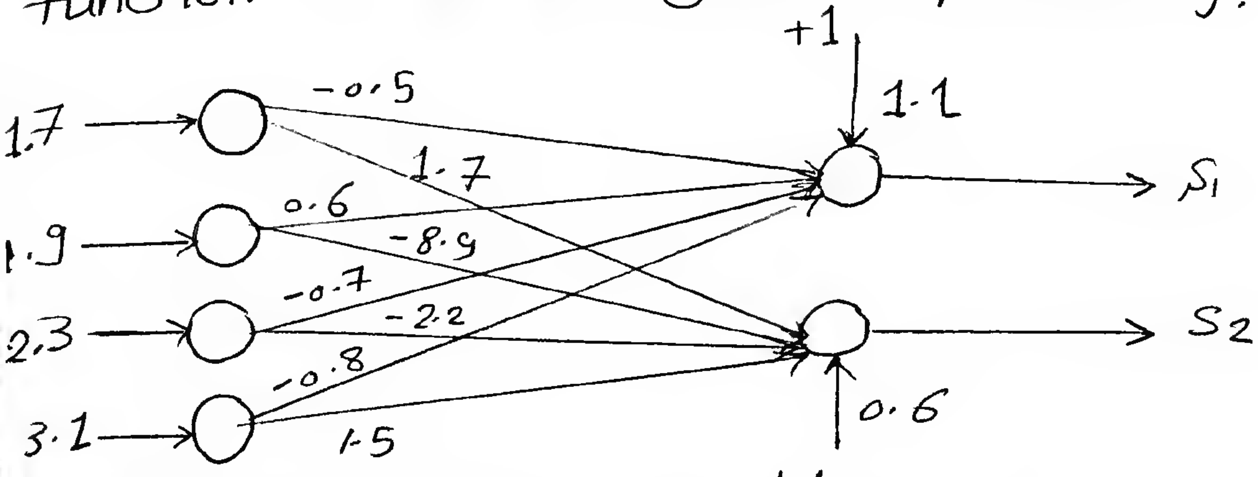

Fig. 1: Neural network for problem 16.

17 Consider the neural network illustrated in Fig.2. All neurons in the hidden and output Layers have a binary sigmoidal function of the form given in Problem.1. Determine the response Signal S.

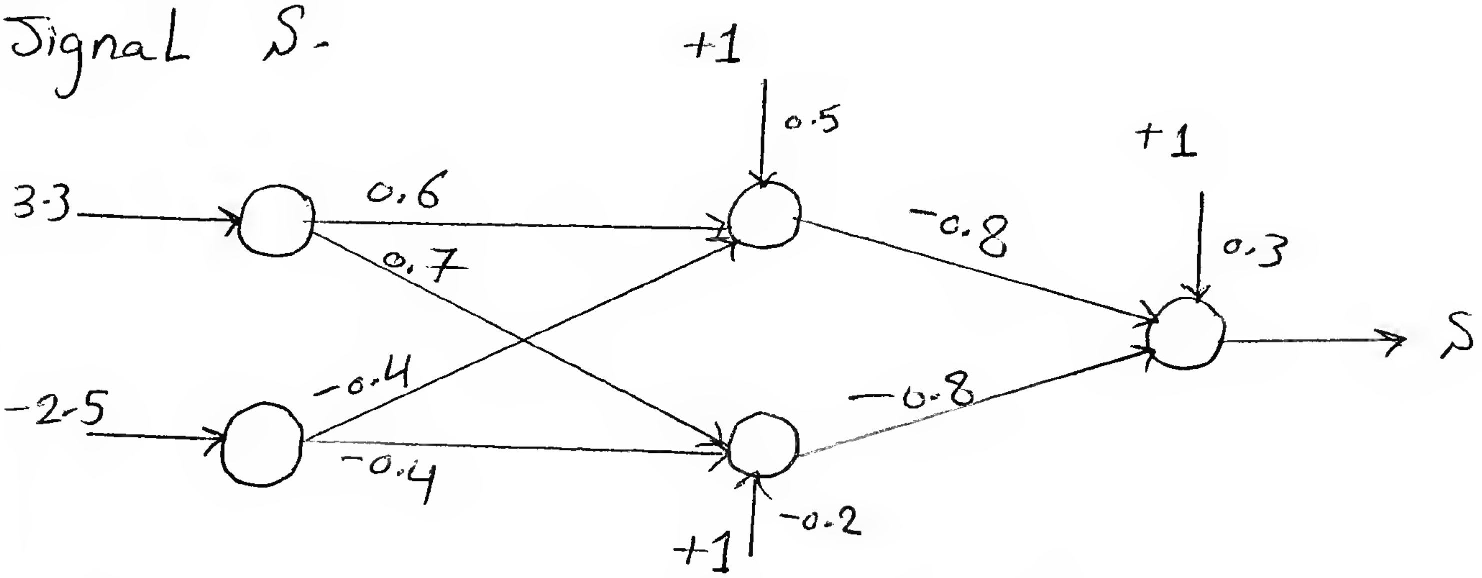

Fig.2: Neural network for problem.17.

18 Consider the neural network illustrated in Fig.3. ALL neurons in the hidden and output layers have a bipolar sigmoidal function of the form given in problem.9. Determine the response signal S.

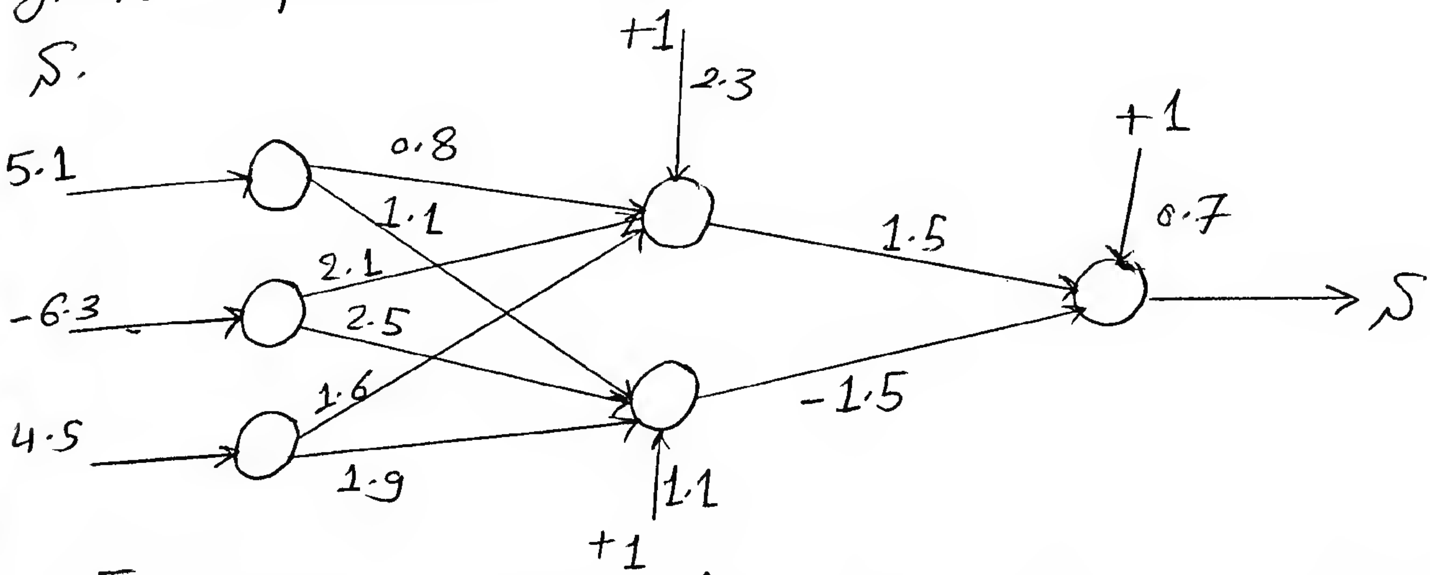

Fig.3: Neural network for problem.18.

19 Consider the neural network illustrated in Fig.4. ALL neurons in the hidden and output layers have abinary Sigmoidal function of the form given in ~~abinar~~ problem.1. If the response signals s_1 and s_2 are required to be equal, verify that the bias weight w_{02} should exceed the bias weight w_{01} by 3.5.

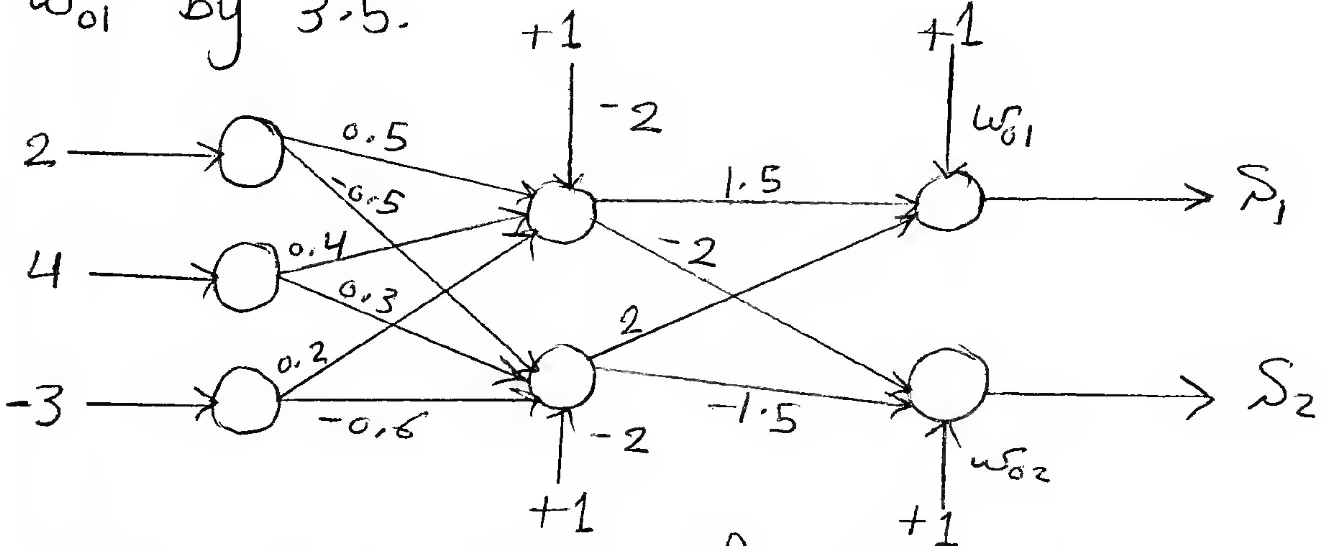

Fig.4: Neural network for Problem.19.

20 Consider the neural network illustrated in Fig.5. ALL neurons in the hidden and output Layers have abinary Sigmoidal function of the form given in problem.1. If the response Signal S is required to be 0.3, find the weight bias, w_0.

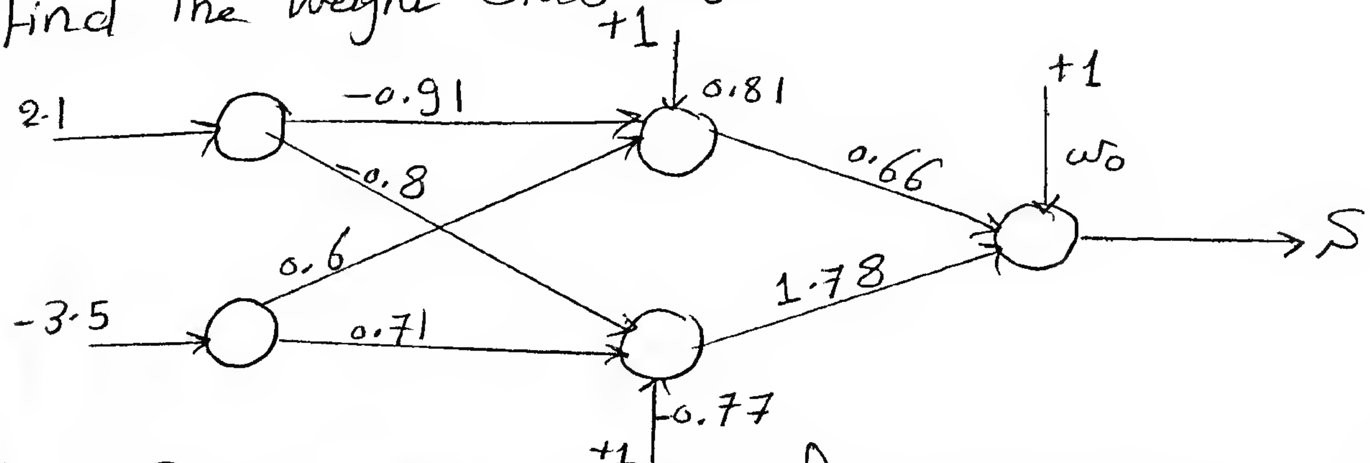

Fig.5: Neural network for problem.20.

21 In problem 20, if the response signal S is required to be -0.3, can you find the corresponding weight bias w_0? Justify.

22 In problem 20, let all neurons in the hidden and output layers have a bipolar sigmoidal function of the form given in problem 9. Find the weight bias w_0 when the response signal S is

a) 0.3

b) -0.3

23 In problem 22, if the response signal S is required to be -1.5, can you find the corresponding weight bias w_0? Justify.

24 In a neural network, the input layer has two neurons N1 and N2 receiving two inputs $x_1 = 3.2$ and $x_2 = 4.4$, respectively. The hidden layer has two neurons N3 and N4 employing a binary sigmoidal function of the form given in problem 1. The output layer has a single neuron N5 employing a bipolar sigmoidal function of the form given in problem 9. The weights (including bias) are:

$w_{13} = 1.5$, $w_{14} = -2$, $w_{23} = -2.2$,

$w_{24} = 3$, $w_{35} = 2$, $w_{45} = -2$,

$w_{03} = 1.5$, $w_{04} = -1.5$, $w_{05} = 1.5$

Determine the response signal, S, of the network.